Advances in Knowledge-Based Systems, Data Science, and Cybersecurity; Research 2 (1) 149-171 Received 10-08-2024; Accepted 28-12-2024; Published 04-01-2025

Performance of Levene's Test With Various Residuals and Correction Factor for Homogeneity of Variance in Single and Factorial Anova-a Simulation Approach

B. Odoi bodoi@umat.edu.gh
Department of Mathematical Sciences, University of Mines and Technology
UMaT, Tarkwa, Ghana

S. Twumasi-Ankrah
Department of Actuarial Science and Statistics, Kwame Nkrumah University of Science and Technology
KNUST, Kumasi, Ghana

B. Afriyie
Department of Mathematics and Science, Ivy Tech Community College
Muncie Campus, U.S.A

S. Samita
Department of Crop Science, University of Peradeniya
Sri-Lanka

Corresponding Author: B. Odoi

Abstract

A prerequisite for the Analysis of Variance (ANOVA) in many academic and practical studies is that the variances of the observed groups are identical, and this is usually called the test of homogeneity of variance (HOV). In literature, the comparison of different HOV tests with respect to their statistical strengths and flaws has been well documented. Regarding their flaws, the comparison of these HOV tests in the framework of factorial ANOVA has not been clearly documented. This research focused on exploring various forms of residuals and introduced a correction factor to boost the performance of Levene's test in single factor and factorial ANOVA. The z_{ij} in the modified Levene's test was obtained based on various forms of residuals $y'_{ij}s$. Again, three different forms of correction factors were also considered in the modification of the test. Using 1000 simulations for every possibility and form of the residual, the coherence of every variant was examined under multiple conditions, including various combinations of factor levels, repetitions, mean variations, and variance deviations to false positives and false negatives. According to the research, merging the application of the absolute residuals of y_{ij} with the correction factor (k/2) was the most effective for the test result, and it held for all situations with factorial and single factor ANOVA. However, using the square of residuals of y_{ij} was superior to the residuals' performance. The statistical likelihood of the test reliably decreases as the number of covariates and the intensity of factors rise, regardless of the number of reproductions, for all types of residuals that have been studied. Again, it was observed that, with the absolute residuals of y_{ij} and correction factor

Citation: Odoi et al. Performance of Levene's Test With Various Residuals and Correction Factor for Homogeneity of Variance in Single and Factorial Anova-a Simulation Approach. Advances in Knowledge-Based Systems, Data Science, and Cybersecurity; Research 2024;2(1):7.

(k/2), the efficiency of Levene's test can be improved, and the problem of the conservative of the test can be dealt with.

Keywords: Residuals, Factorial ANOVA, Correction factor, False positives, False negatives

1. INTRODUCTION

A crucial and difficult method used across many fields is testing the Homogeneity of Variance (HOV) in factorial ANOVA [1]. In more complex factorial ANOVA, the statistical methods for the factors affecting the cell variances are typically approximate and imprecise [2–5]. A number of the HOV tests have been developed to solve problems with Levene's test yet no better solution has been suggested especially with factorial ANOVA [4–6]. Levene's test is one of the most widely used methods for evaluating HOV across a range of fields [7–13]. Many researchers apply these tests in several disciplines to determine whether the variances among groups are homoskedastic. However, conflicting findings of false positives and false negatives in using the Levene's test were documented in most of the research examined, particularly in the context of factorial ANOVA [14–16]. This presents many unanswered issues that must be dealt with when evaluating HOV in factorial ANOVA [11–17]. Equal cell frequencies are a requirement for the [18] process, and picking the right Levene test requires careful consideration. According to [4, 5, 19], the majority of HOV tests, particularly Levene's and Bartlett's tests, do not extend the impact of independence on the variation in the sample of numerous variables in factorial ANOVA. Levene's test ought to be utilized cautiously, even though false negatives are largely lost, nevertheless, it has better results for false positives error [20–23]. Regarding evaluating HOV, the following knowledge voids were found in literature: Statistical power efficiency and false-positive control of the HOV test have yielded conflicting findings in studies [5, 7–9, 12, 13, 22–29], claimed that while Levene's test is reliable for analyzing identical variance, various kinds of residuals may be used to enhance Levene's test when analyzing HOV in factorial ANOVA. The study addresses issues with Levene's conservativeness tests by looking into (i) the existing residual formulations used for evaluating HOV and (ii) incorporating a correction factor to solve these issues in factorial ANOVA. The study deployed this concept using a simulation approach. The foundation of this strategy involves changing the repetitions' numbers and groups. The ensuing particular goals were executed to achieve this objective: Levene's test's coherence in single and factorial ANOVA was examined, as well as its effectiveness in testing HOV as stated in this research for single and factorial ANOVA. A modified Levene's test that is non-conservative was proposed for testing HOV in single and factorial ANOVA.

1.1 RELATED WORKS

Several techniques have lately been used to assess the uniformity of variance in factorial ANOVA; however, the statistical power of these studies is low Jayalath et al., 2017. According to [6, 30–34], Levenes' test successfully controlled for false positives if the population is distributed normally. To limit the false positives and non-normal data for univariate designs, [24], changed Levene's and Bartlett's tests based on the Hines test (W_H) by employing the bootstrap test built on the proportion of mean absolute deviation. His research and recommendations, however, ran counter to those of [25]. Although the O'Brien test was again advised by [25], as one of the best methods that maintain adequate false positives command, it was found that it did badly regarding statistical power. The Kruskal-Wallis test is generalized across all factorial models with data that is distinct using a permutation test method proposed by [35], flexibly. [5], investigated the effectiveness of Bartlett's and Levene's tests for evaluating HOV in a one-way ANOVA with various repetitions and groups. It was determined that to obtain acceptable false negatives, at least 30 repetitions are required.[23], proposed Bartlett's test, which utilized various residuals to concurrently lower false positives and false negatives with fewer repetitions and levels. The test, however, was found to be cautious with a rise in covariate. The Goal of the Study aimed to investigate the reliability and effectiveness of Levene's test in single and factorial ANOVA by examining various residuals and incorporating a correction factor to aid in the joint reduction of false-positive and false negatives mistakes. The following were the study's inquiries;

- i will changing the test's level count and regularity make it better?
- ii can we investigate the various residual forms when using Levene's test to assess HOV while changing the uniformity and variety of levels?
- iii can there be an introduction of a correction factor to reduce false positives and false negatives simultaneously in testing HOV in factorial ANOVA?

2. THEORETICAL FRAMEWORK AND METHODOLOGY

2.1 METHODOLOGY

Levene's test as well as a single and factorial design were the techniques used in this research.

2.1.1 Levene's Test

Levene's technique is employed to determine if k samples have equal variances. The usual F- ratio in single-factor ANOVA served as the basis for Levene's test with the j^{th} observation in i^{th} group, y_{ij} replacing its total departure from its group means, $z_{ij} = |y_{ij} - \bar{y}_i|$ where $\bar{y}_i$ is the estimated measuring point of the i^{th} group. Given that a variable y coupled with the overall quantity of data n divided into k groups, where n_i is how many times the data was replicated for the i^{th} group, Consequently, Levene's test statistic is given as

$$W = \frac{(n-k)\sum_{i=1}^{k} n_i(\bar{z}_{i.} - \bar{z}_{..})^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(z_{ij} - \bar{z}_{i.})^2} \quad (1)$$

where $\bar{z}_{i.} = \frac{\sum_{j=1}^{n_i} z_{ij}}{n_i}$, $\bar{z}_{..} = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i} z_{ij}}{n}$ and $z_{ij} = |y_{ij} - \bar{y}_i|$

Levene's test rejects the null hypothesis of common variance at the significant$k$ threshold of α if $WF_{\alpha,k-1,n-k}$ where $F_{\alpha,k-1,n-k}$ is the F- distribution's highest critical number.

$$\textbf{Note: } z_{ij} = |y_{ij} - \bar{y}_i|, \quad (2)$$

z_{ij} was obtained using the individual categories' means.

2.1.2 Limitation of the Traditional Levene's Test

Levene's test has been recommended to check for the uniformity of variations because it performs better to false positives than false negatives. An investigation into the test has demonstrated its conservatism. [5, 25, 30]. Again, as the number of variables rises in a factorial ANOVA, the estimated strength of the test decreases.

2.1.3 Suggested Solution to Levene's Test

A method, which involved using various types of residuals along with proper correction factors, was suggested to increase statistical power and decrease false positives to an insignificant number.

2.1.4 Single and Factorial Trial Design

Single Factor Design

We typically look into how an element affects an answer. So, using various proportions of the component, we run an experiment. These tests are frequently referred to as single-factor tests. The ability to identify non-linear implications once the variable of interest is continuous is one of the benefits of a single-factor design with a minimum of two layers. [33, 36, 37].

It is, however, one of the minimum effective methods for carrying out tests. This is so that one cannot investigate how multiple variables interact to affect the response parameter of interest when only one factor is varied at a time. It has been demonstrated analytically that analyzing the impact of a single variable without concurrently varying several other variables can be deceptive [38].

A single-factor design paradigm is provided as;

$$X_{ij} = \mu + \tau_i + \varepsilon_{ij} \tag{3}$$

Where μ is the grand mean, τ_i is the effect of i^{th} treatment and ε_{ij} is the random error or residual.

2.1.5 Factorial trials

In a factorial ANOVA, we examine the impacts of multiple factors at once. The treatments are referred to as factors, and a factor may have multiple degrees [33]. The main goal of factorial trials is to examine the interaction effects of one component at various degrees of the other factor. All possible combos of the amounts of various variables make up a factorial trail. [34, 39]. In this study, a two-factor and three-factor, Completely Randomized Design (CRD) with higher order factorial trials were executed. Taking into account a factorial ANOVA consists of A and B factors. If there exist *a* and *b* levels, accordingly, for A and B. The statistical approach for an ANOVA assuming the method used is a CRD with r repetitions is given as,

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \tag{4}$$
$$i = 1, 2, \ldots, a$$
$$j = 1, 2, \ldots, b$$
$$k = 1, 2, \ldots, r$$

Where;

y_{ijk} = the k^{th} observation on i^{th} level of A and j^{th} level of B, μ = grand mean, α_i = effect of the i^{th} level of A, β_j effect of the j^{th} level of B, $(\alpha\beta)_{ij}$ =interaction effect of i^{th} level of factor A and j^{th} level of factor B, ε_{ijk} random error.

An investigation using three factors—A, B, and C—in a factorial design is another option to examine. For factors A, B, and C, respectively, let the number of levels be. The statistical model for an analysis of variance (ANOVA) is of the form:

$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{iik} + \varepsilon_{ijkl} \tag{5}$$
$$i = 1, 2, \ldots, r$$
$$j = 1, 2, \ldots, a$$
$$k = 1, 2, \ldots, b$$
$$l = 1, 2, \ldots, c$$

Where;

y_{ijkl} = the observation on j^{th} level of A, k^{th} level of B and l^{th} level of C in i^{th} block, μ = grand mean, ρ_i = effect of i^{th} block α_j = effect of j^{th} level of factor A, β_k = effect of k^{th} level of factor B, γ_l = effect of l^{th} level of factor C, $(\alpha\beta)_{jk}$ = interaction effect of j^{th} level of factor A and k^{th} level of factor B, $(\alpha\gamma)_{jl}$ = interaction effect of j^{th} level of factor A and l^{th} level of factor C, $(\beta\gamma)_{kl}$ = interaction effect of k^{th} level of factor B and l^{th} level of factor C, $(\alpha\beta\gamma)_{jkl}$ = interaction effect of j^{th} level of factor A , k^{th} level of factor B and l^{th} level of factor C , andε_{ijkl} random error.

2.1.6 Basic Assumptions for m^k factorial ANOVA

k represents the number of components, having m levels. Everything regarding the layout is completely randomized.

The covariates are constant.

1. The normalcy premise is supported.
2. Over the selection of component values, the reaction is essentially linear.
   [40].

2.2 THEORETICAL FRAMEWORK

The methods, as well as the empirical and mathematical concepts behind the study, are covered in the conceptual structure of this research.

2.2.1 Single factor ANOVA

The single-factor ANOVA's residual term is shown in equation (6) as:

$$\mu_{ij} = X_{ij} - \widehat{X}_{ij} \quad (6)$$

Where $X_{ij} = j^{th}$ response value for the i^{th} level of factor A

$\widehat{X}_{ij} = j^{th}$ fitted value for the i^{th} level of factor A

2.2.2 Two-factor factorial ANOVA

The two-factor factorial ANOVA residual term shown in equation (7) as:

$$\mu_{ijk} = X_{ijk} - \widehat{X}_{ijk} \quad (7)$$

Where $X_{ijk} = k^{th}$ response value for the i^{th} level of factor A and the j^{th} level of factor B.

$\widehat{X}_{ijk} = k^{th}$ fitted value for the i^{th} level of factor A and the j^{th} level of factor B

2.2.3 Three factor factorial ANOVA

The three factor factorial ANOVA residual term shown in equation (8) as:

$$\mu_{ijkl} = X_{ijkl} - \widehat{X}_{ijkl} \quad (8)$$

2.2.4 Utilization of various residuals and correction variables

The residuals are acquired using the subsequent techniques or processes.:

- Randomly produced normal data was used.
- Equation (3) was used to fit the model to the normal data created in stage 1 and fit the model.
- The residuals $\mu_{ij} = X_{ij} - \widehat{X}_{ij}$ were obtained from the fitted model and μ_{ij} were used as y_{ij} to determine if the hypotheses of equal variations were infringed, 1000 simulations were performed according to each of 60 scenarios and their p-values were calculated. subsequently, examining false-positive and false negatives. Step 3 also produced several residuals in different formats, including the absolute, square, and square root of the absolute residuals.

- This was employed to perform 1000 simulations under 60 scenarios and acquire the p-values (ρ_i) to determine if the hypotheses were actually broken or not. By doing so, you can verify the false positives and false negatives.
- The percentage of occurrences in the test falsely determined that the statistic possessed inconsistent variances was used to compute false positives a $\alpha = 0.05$.
- false negatives show the percentage of occurrences the test was unable to reach its conclusion. that the values of the various distributions were not equivalent.

2.2.5 The residuals from Levene's test in their absolute, square root of absolute, and squared versions

The results of Levene's test residuals for a single-factor, two-factor factorial, and higher-order component ANOVA were shown in their absolute, square root of absolute, and squared versions. The trend was non-traditional when the square root of the absolute residuals was taken into account, and this caused the false positives and false negatives to rise to unacceptable levels as the number of repetitions increased. As a result, it was eliminated from the research.

2.2.6 A suggested Levene's test without a correction factor

The method for the suggested Levene's Test is described in this part. The absolute residuals, the square of the residuals, and the square root of the absolute residuals were all used to evaluate the various residuals that Levene's test suggested.

The structure of the model was initially fitted using Equation (3), and the residuals were calculated using Equation (6).

Then y_{ij} in Equation (1) was defined as suggested in different forms, absolute, square, and square root of the absolute of the residual as

$$y_{ij} = \left|u_{ij}\right| \tag{9}$$

$$y_{ij} = (u_{ij})^2 \tag{10}$$

$$y_{ij} = \sqrt{\left|u_{ij}\right|} \tag{11}$$

Now,

$$z_{ij} = \left|y_{ij} - \widehat{y}_{i.}\right| \tag{12}$$

Using z_{ij} in above different forms equation (1), *W the* statistic was computed.

2.3 LEVENE'S TEST WITH CORRECTION FACTOR: PROPOSED MODIFICATION

The suggested Levene's Test technique is described in this part. By drastically reducing false positives to an extremely minuscule value and increasing the statistical strength of the test with fewer groups and repetitions, a correction factor was introduced. The correction factors considered were k, k^2 and (k/2). A pattern known as the F- distribution, however, (k/2) happens to be the best because it keeps false positives asymptotically at low values as statistical strength rises. Again, as a consequence of the simulation findings, the correction factor was introduced. As the number of groups and factors increased, the statistical strength of the tests using the different residual forms decreased noticeably. Where k is the number of groups. In this instance, the correction factor was incorporated to keep and lower false positives to an incredibly small number while also boosting the statistical strength of the tests. The essence of the correction factor introduced was based on the research done by [29].

(k/2) of absolute residuals Levene's Test.

$$W^{**} = \left\{ \frac{(n-k)\sum_{i=1}^{k} n_i (\bar{z}_{i.} - \bar{z})^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}(z_{ij} - \bar{z}_{i.})^2} \right\} \left(\frac{k}{2}\right) \tag{13}$$

where k represents the number of groups, $z_{ij} = \left| y_{ij} - \widehat{y}_{i.} \right|$, $\bar{z}_{i.} = \frac{\sum_{j=1}^{n_i}(z_{ij})}{n_i}$ and $\bar{z}_{..} = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i}(z_{ij})}{n}$

The altered Levene's test denies the null hypothesis of same variances at the significant threshold of α if $W^{**} \succ F_{\alpha,k-1,N-K}$, where $F_{\alpha,k-1,N-K}$ is the upper critical value of the F- distribution.

Where $N = nk$ and $K = k^2$

Note: z_{ij} was calculated from the deviation of the observed value and the fitted value of the respective groups.

2.3.1 Single factor ANOVA

According to equation (4), the residual term of single-factor ANOVA in Equation (2) is given as

$$\mu_{ij} = X_{ij} - \widehat{X}_{ij} \tag{14}$$

Where $X_{ij} = j^{th}$ response value for the i^{th} level of factor A, $\widehat{X}_{ij} = j^{th}$ fitted value for the i^{th} level of factor A

2.3.2 Two Factor factorial ANOVA

As mentioned in equation (5), a two-factor factorial ANOVA, the residual term of two-factor factorial ANOVA is given as

$$\mu_{ijk} = X_{ijk} - \widehat{X}_{ijk} \tag{15}$$

Where $X_{ijk} = k^{th}$ response value for the i^{th} level of factor A and the j^{th} level of factor B, $\widehat{X}_{ijk} = k^{th}$ fitted value for the i^{th} level of factor A and the j^{th} level of factor B

3. SIMULATION

Simulated data produced by the statistical program R were used in the research [41]. The effectiveness of Levene's test was assessed using the produced data. Simulations were performed under various situations of repetitions and levels for each, according to the goals. For each situation, 1000 models were run, and data were produced to fit the normal distribution. The following assumptions served as the basis for the simulation:

i various levels of factors

ii various numbers of replications

iii mean distinction

iv change in variance

v comparing different variations that are pertinent to the gap disparities

Notably, the selection of the different modeling techniques followed the literature [5, 24, 25, 33, 42]. The data framework for the simulation in this research was, however, adapted from works by [5, 40], for both the single-factor and factorial ANOVA.

3.1 FALSE POSITIVES AND FALSE NEGATIVES

The false positives and false negatives centered on the percentage of times each test wrongly determined that the distribution had uneven variations at $\alpha = 0.05$, false positives were computed in this research. The consideration of $\alpha = 0.05$ was for higher accuracy. The assumption is that the result will be constant even with various values because it is considered that accuracy and stability do not vary with a variation in α values. False negatives show the percentage of occurrences each test declined to conclude that the values of the various distributions were not identical.

4. RESULTS AND DISCUSSIONS

Single-factor ANOVA trials are intended to offer a lot of valuable data in evaluating a specific issue, so by using such a design, one can determine which independent variable impacts the behavior. It provides sufficient outcomes since the methodology is simple. In a single-factor ANOVA, the investigation depends on the impact of the independent variable across different instances.

A comparison of Levene's test results using standard, adapted residuals and the correction factor for single-factor, two-factor, and higher-order factorial ANOVA.

The contrast of the Levene test false-positives and false-negatives using the traditional residuals, squared residuals, absolute residuals, and a correction factor is discussed in this part.

4.0.1 Analysis

With more levels and uniformity, the false positives in the study for all instances improved, supporting the fact that factorial ANOVAs with one, two, and more factors have statistically significant effects. Although there was an improvement in false positives as the number of replications increased for the single-factor example when using the traditional Levene's test. Despite having 30 replications, the false-positive was 0.053, which is higher than the meaningful criterion set at 5%. This shows that the test is underperforming when using traditional approaches. Instead, using the adapted Levene's test with various residuals and a correction factor improved the outcomes with a rise in the number of levels and stability. As indicated in TABLE 1 the findings for the traditional Levene's method are as follows: for levels of three (3) at five (5) trials, the residual is 0.053, the squared residual is 0.052, the absolute residual is 0.051, and the residual with correction factor is 0.049. It suggests that the test is more effective when they are conducted using the adapted Levene's test with a correction factor. With levels of 4, and 5, the modified Levene's test's effectiveness improved steadily. For instance, at levels of 5, the false-positives for traditional Levene's test were 0.056, 0.048 for modified residual, squared residuals are 0.047, absolute residuals are 0.044, and the correction factor is 0.027. As a consequence, the findings aligned with both a rise in levels and a corresponding increase in duplicates. The findings for all occurrences of false negatives improved when the number of replications was increased. The test proved conservative with a rise in levels for both the traditional residuals and even the modified residuals. However, modifying Levene's test with the correction factor reduced false negatives to a substantially small value to solve the problem of conservative. For instance, at repetitions of 5 for levels of 3, the false negatives for convention Levene's test were 0.951, 0.812 for residuals, 0.612 for squared residuals, 0.511 for absolute residuals, and 0.281 for correction factor. At repetitions of 5 for levels of 4, the false-negatives for convention Levene's test were 0.915, 0.856 for residuals, 0.615 for squared residuals, 0.515 for absolute residuals, and 0.321 for correction factor. Also, at a repetition of 5 for levels of 5, the false-negatives for convention Levene's test were 0.862, 0.861 for residuals, 0.631 for squared residuals, 0.581 for absolute residuals and 0.378 for correction factor as shown in TABLE 4.36. When analyzing a two-factor factorial example, the traditional Levene's test, the findings showed that there was an improvement in false positives as the number of replications increases. However, in 30 repetitions, the false-positive rate was 0.071 at levels of 3 for each component, higher than the significance threshold set at 5%, indicating that the traditional test is not performing effectively. On the other hand, applying the updated residuals along with a correction factor produced more consistent, better-leveled results. Thus, for levels of three (3) for each factor at 5 repetitions, the results for the traditional Levene's is 0.087, the residual is 0.053, the squared residual is 0.045, the absolute residual is 0.049, and with the correction factor is 0.042 as shown in TABLE 1. This indicates that using the modified Levene's test with a correction factor improves the effectiveness of the test. With levels of 4, and with levels 5, there is a constant increase in the modified Levene's test's effectiveness when using the residuals and correction factor versions. Therefore, the findings are consistent with both a rise in levels and a corresponding increase in repetitions. The findings for all occurrences of false negatives improved when the number of replications was increased. The test was cautious with more levels when using traditional and even modified residuals. However, the modified Levene's test with a correction factor reduced false negatives to a substantially small value to solve the problem of conservative. For instance, at repetitions of 5 at levels of 3 for each factor, the false-negatives for traditional Levene's test were 0.921, 0.812 for residuals, 0.532 for squared residuals, 0.423 for absolute residuals, and 0.401 for correction factor. At repetitions of 5 for levels of 4, the false negatives for traditional Levene's test were 0.815, 0.785 for residuals,

Table 1: False Positives of Levene's Test with Various Residuals and Correction Factor for single factor, two-factor factorial and higher order factorial ANOVA

Levene's Test - False Positives												
Single-factor ANOVA												
	Residuals			Absolute Residuals			Square Residuals			Correction Factor		
Repli-cations	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor
3	0.078	0.081	0.082	0.068	0.071	0.078	0.070	0.078	0.076	0.051	0.056	0.060
4	0.052	0.077	0.079	0.050	0.061	0.069	0.048	0.071	0.071	0.051	0.054	0.055
5	0.053	0.057	0.050	0.051	0.054	0.051	0.052	0.062	0.043	0.049	0.051	0.050
8	0.042	0.052	0.050	0.039	0.049	0.044	0.041	0.046	0.052	0.032	0.035	0.024
10	0.040	0.048	0.055	0.041	0.044	0.048	0.039	0.047	0.051	0.026	0.027	0.021
15	0.032	0.050	0.047	0.033	0.046	0.045	0.030	0.049	0.042	0.020	0.019	0.022
20	0.043	0.046	0.046	0.032	0.041	0.043	0.040	0.043	0.041	0.019	0.021	0.020
25	0.045	0.038	0.045	0.031	0.043	0.041	0.041	0.043	0.041	0.015	0.017	0.017
30	0.042	0.043	0.042	0.029	0.040	0.039	0.039	0.035	0.043	0.012	0.013	0.016
Two-factor factorial ANOVA												
	Residuals			Absolute Residuals			Square Residuals			Correction Factor		
	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor
3	0.078	0.081	0.082	0.070	0.071	0.072	0.074	0.071	0.073	0.052	0.057	0.055
4	0.052	0.077	0.079	0.051	0.072	0.071	0.047	0.067	0.064	0.048	0.053	0.054
5	0.043	0.057	0.050	0.049	0.057	0.067	0.045	0.047	0.049	0.042	0.048	0.049
8	0.042	0.050	0.050	0.039	0.040	0.048	0.038	0.041	0.049	0.034	0.035	0.040
10	0.040	0.055	0.055	0.032	0.035	0.046	0.038	0.038	0.050	0.022	0.030	0.036
15	0.032	0.050	0.047	0.029	0.031	0.041	0.028	0.044	0.039	0.019	0.027	0.029
20	0.043	0.046	0.046	0.022	0.028	0.039	0.034	0.039	0.043	0.017	0.021	0.027
25	0.045	0.038	0.045	0.020	0.031	0.028	0.041	0.027	0.042	0.018	0.019	0.022
30	0.042	0.043	0.042	0.019	0.024	0.029	0.028	0.032	0.038	0.017	0.020	0.024
Higher order factorial ANOVA												
	Residuals			Absolute Residuals			Square Residuals			Correction Factor		
	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor
3	0.075	0.079	0.081	0.071	0.072	0.077	0.071	0.074	0.071	0.050	0.053	0.061
4	0.074	0.076	0.081	0.061	0.061	0.067	0.069	0.071	0.078	0.051	0.054	0.056
5	0.062	0.071	0.068	0.058	0.060	0.062	0.061	0.068	0.069	0.049	0.050	0.055
8	0.050	0.047	0.060	0.044	0.041	0.059	0.049	0.042	0.059	0.031	0.045	0.047
10	0.046	0.043	0.049	0.042	0.039	0.054	0.042	0.039	0.048	0.028	0.032	0.042
15	0.044	0.041	0.048	0.039	0.044	0.042	0.040	0.032	0.044	0.022	0.029	0.038
20	0.043	0.040	0.044	0.035	0.039	0.039	0.039	0.042	0.042	0.024	0.026	0.034
25	0.045	0.044	0.047	0.032	0.037	0.032	0.022	0.044	0.039	0.020	0.025	0.028
30	0.043	0.041	0.046	0.029	0.038	0.040	0.021	0.040	0.044	0.021	0.024	0.023

0.556 for squared residuals, 0.681 for absolute residuals, and 0.321 for correction factor. Also, at repetitions of 5 for levels of 5, the false-negatives for convention Levene's test were 0.812, 0.801 for residuals, 0.661 for squared residuals, 0.691 for absolute residuals, and 0.391 for correction factor as shown in TABLE 2. This research demonstrates that applying a correction factor to Levene's test decreases false negatives by a considerable amount, increasing the test's statistical accuracy. These outcomes were the same for the factorial ANOVA with higher factors. it should be noted that the results from the squared residuals were as good as that of the absolute residuals.

4.0.2 Correction factor (k/2).

For the case of single-factor ANOVA, with false positives, Levene's test improved much better with a lesser number of levels, and consistency. The results of the analysis revealed that for the correction factor *(k/2)*, with 3 levels type, I error decreased from 0.051 to 0.012 for Levene's test, which was thought-provoking and amazing even though with a lesser number of levels and repetitions. Statistically, when the number of replications increases then the power of most of the test statistics becomes better at the expense of false positives and vice versa. For the two-factor factorial ANOVA case, with false positives, the modified Levene's Test with correction factor *(k/2)* had a tremendous performance, reduced drastically from 0.059 to 0.019 as shown in TABLE 5a. Even with 3 or fewer replications at a fixed number of levels, the analysis's results essentially plateaued.

For higher-order factorial ANOVA, false positives remain the same for all. false-positives were reduced from 0.060 to 0.020 using the correction factor *(k/2)*. This result seems to be the best performance with increased factorial ANOVA. In summary, one may vigorously conclude that the modified Levene's and Levene's tests *(k/2)* with correction factor control false-positives at a faster rate. However, the results from the correction factor *(k/2)* were the best. Based on many simulation studies, it is quite obvious that false positives reduce when the number of replications increases with an increase in the number of levels, however, it is quite rare for situations where false positives reduce and the power of the test increases. This phenomenon happened with the introduction of the correction factor. Thus, with a lesser number of replications and lesser number of levels the modified Levene's test is performing better even with higher-order factor ANOVA Cases. For false negatives with single-factor ANOVA, the results of the analysis revealed that there was a decrease in false negatives from 0.28 to 0. 10 across levels as shown in TABLE 5b.

4.0.3 Using correction factors k and k^2

Considering false-positives, for single factor ANOVA using correction factor k with 3 levels was 0.031 for k^2 was 0.46 for Levene's test with an increase in the number of replications. For two–factor factorial ANOVA with 3 levels each, the results for using correction factor, k^2were 0.038 and 0.54 for Levene's tests. Also, for higher-order factorial ANOVA the results for using correction factors, k and k^2 were 0 0.041 and 0.049 for Levene's test. For false negatives, for a single factor ANOVA using correction factor k and k^2 with 3 levels was 0.47 and 0.54 for Levene's test with an increase in the number of replications. With two–factor factorial ANOVA with 3 levels each, the results for using correction factor, k and k^2were 0.51,0.59 for Levene's test. Also, for higher-order factorial ANOVA the results for using correction factors, k and k^2 were 0.61 and 0.73 for Levene's test. This shows that their false positives were reduced but at the expense of false negatives. Thus,

Table 2: False Negatives of Levene's Test with various Residuals and Correction Factor for single factor, two-factor factorial and higher order factorial ANOVA

Levene's Test - False Negatives												
Single-factor ANOVA												
	Residuals			Absolute Residuals			Square Residuals			Correction Factor		
Replications	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor
3	0.891	0.911	0.861	0.791	0.717	0.733	0.791	0.747	0.743	0.381	0.402	0.453
4	0.861	0.871	0.842	0.761	0.621	0.601	0.711	0.721	0.821	0.311	0.410	0.421
5	0.812	0.785	0.801	0.511	0.515	0.581	0.612	0.615	0.631	0.281	0.321	0.378
10	0.731	0.741	0.795	0.531	0.521	0.501	0.611	0.501	0.651	0.241	0.291	0.321
15	0.615	0.642	0.671	0.325	0.433	0.321	0.521	0.673	0.512	0.191	0.251	0.302
20	0.531	0.615	0.561	0.211	0.321	0.417	0.421	0.411	0.442	0.121	0.192	0.241
25	0.461	0.501	0.521	0.214	0.331	0.312	0.361	0.390	0.467	0.110	0.151	0.210
30	0.411	0.451	0.431	0.201	0.191	0.291	0.211	0.311	0.451	0.100	0.121	0.200
Two-factor factorial ANOVA												
	Residuals			Absolute Residuals			Square Residuals			Correction Factor		
	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor
3	0.861	0.871	0.891	0.742	0.712	0.761	0.761	0.771	0.791	0.461	0.443	0.431
4	0.841	0.853	0.877	0.712	0.691	0.741	0.641	0.653	0.677	0.442	0.411	0.411
5	0.812	0.856	0.861	0.423	0.681	0.691	0.532	0.556	0.661	0.401	0.321	0.391
10	0.731	0.847	0.781	0.321	0.411	0.672	0.431	0.547	0.581	0.302	0.321	0.323
15	0.615	0.662	0.712	0.356	0.392	0.662	0.323	0.462	0.412	0.240	0.251	0.301
20	0.531	0.532	0.467	0.291	0.218	0.542	0.233	0.432	0.267	0.221	0.221	0.321
25	0.461	0.490	0.475	0.212	0.219	0.451	0.364	0.390	0.375	0.167	0.192	0.291
30	0.411	0.412	0.441	0.192	0.181	0.392	0.212	0.212	0.221	0.132	0.152	0.201
Higher order factorial ANOVA												
	Residuals			Absolute Residuals			Square Residuals			Correction Factor		
	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor	3 levels for the factor	4 levels for the factor	5 levels for the factor
3	0.801	0.812	0.854	0.791	0.712	0.743	0.701	0.712	0.754	0.521	0.531	0.541
4	0.791	0.789	0.831	0.692	0.691	0.692	0.681	0.689	0.711	0.493	0.464	0.501
5	0.621	0.613	0.714	0.601	0.547	0.543	0.521	0.513	0.674	0.471	0.459	0.452
10	0.692	0.645	0.689	0.491	0.503	0.442	0.492	0.525	0.549	0.421	0.421	0.422
15	0.671	0.613	0.681	0.331	0.531	0.543	0.471	0.513	0.581	0.313	0.431	0.411
20	0.512	0.491	0.613	0.391	0.301	0.432	0.312	0.391	0.413	0.213	0.332	0.331
25	0.463	0.391	0.578	0.413	0.290	0.302	0.263	0.241	0.378	0.201	0.270	0.252
30	0.381	0.331	0.491	0.192	0.201	0.291	0.211	0.231	0.291	0.131	0.210	0.212

in using the inflated factors, $k$and k^2 the statistical power of the tests reduces. Again, the pattern of change in terms of the distribution did not give a better fit of the F- distribution. However, there was a finding that using (k/2) false positives reduces significantly to a small value while false negatives

Table 5a: False positives for 2*3 factorial trials with three repetitions deploying various forms of residuals and correction factor

Levene's Test				
Source	Resi.	Absolute of Resi.	Square of Resi.	Correction factor
Factor A	0.048	0.040	0.049	0.030
Factor B	0.050	0.037	0.047	0.024
Interact. effects	0.045	0.049	0.042	0.018

Table 5b: false negatives for Levene's test for 2*3 factorial trials with three repetitions deploying various forms of residuals and correction factor

Levene's Test				
Source	Resi.	Absolute of Resi.	Square of Resi.	Correction factor
Factor A	0.22	0.18	0.21	0.20
Factor B	0.28	0.26	0.23	0.17
Interact. effects	0.25	0.19	0.22	0.13

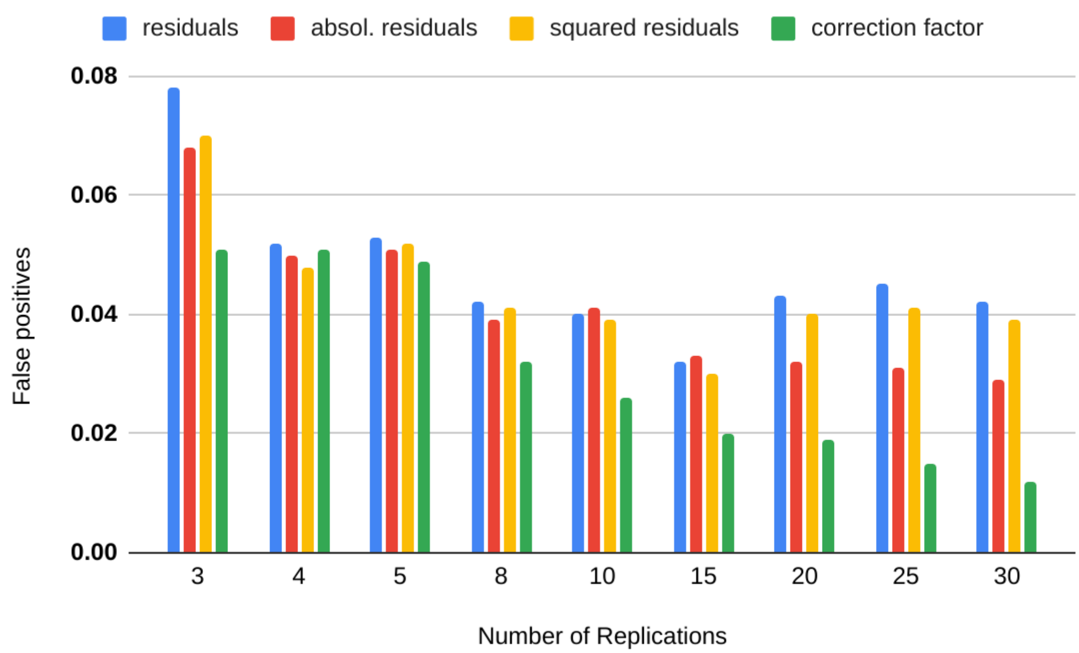

Figure 1: Comparison of false positives for Single-factor ANOVA

also reduce and thereby increase the statistical power of the tests, and also the pattern of change in terms of the distribution mimics that of the F-distribution.

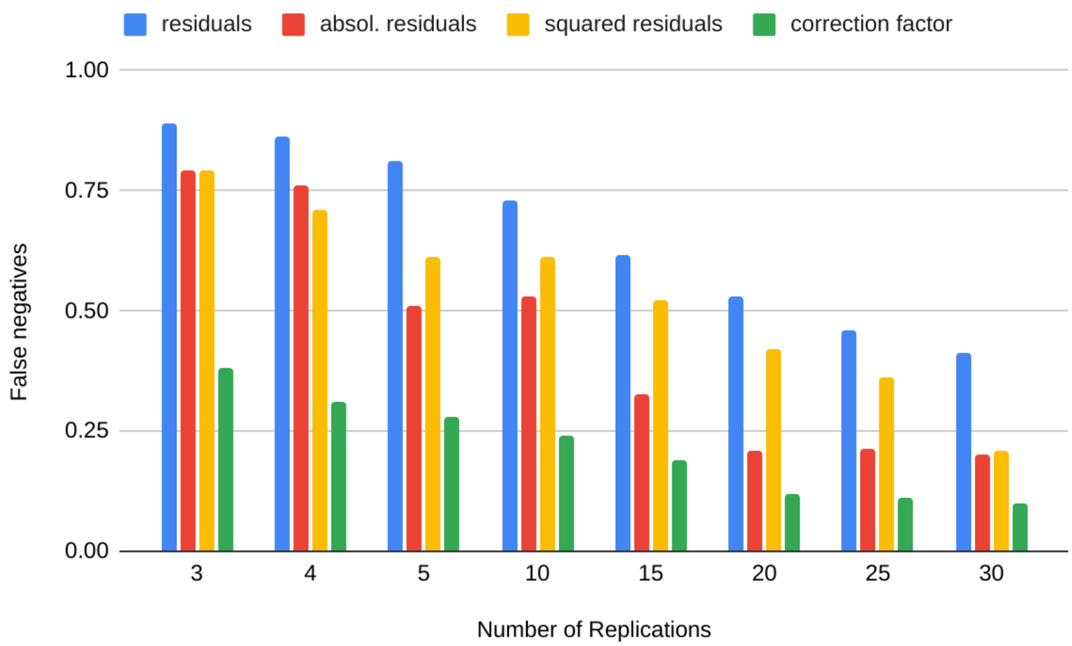

Figure 2: Comparison of false negatives for single factor ANOVA

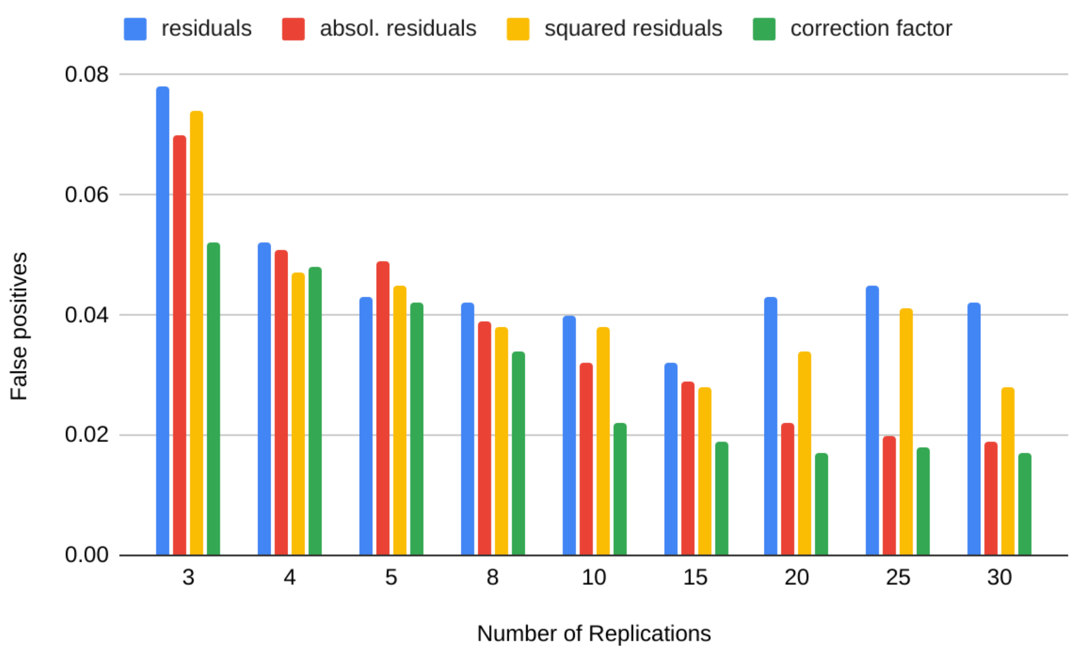

Figure 3: Comparison of false positives for two-factor factorial ANOVA

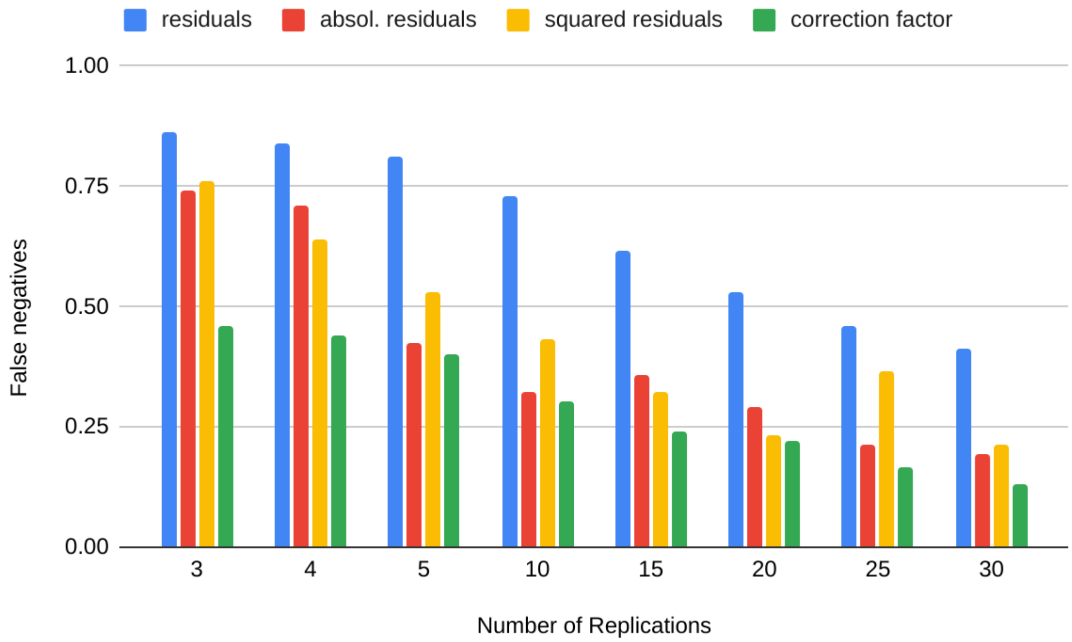

Figure 4: Comparison of false negatives for two factor factorial ANOVA

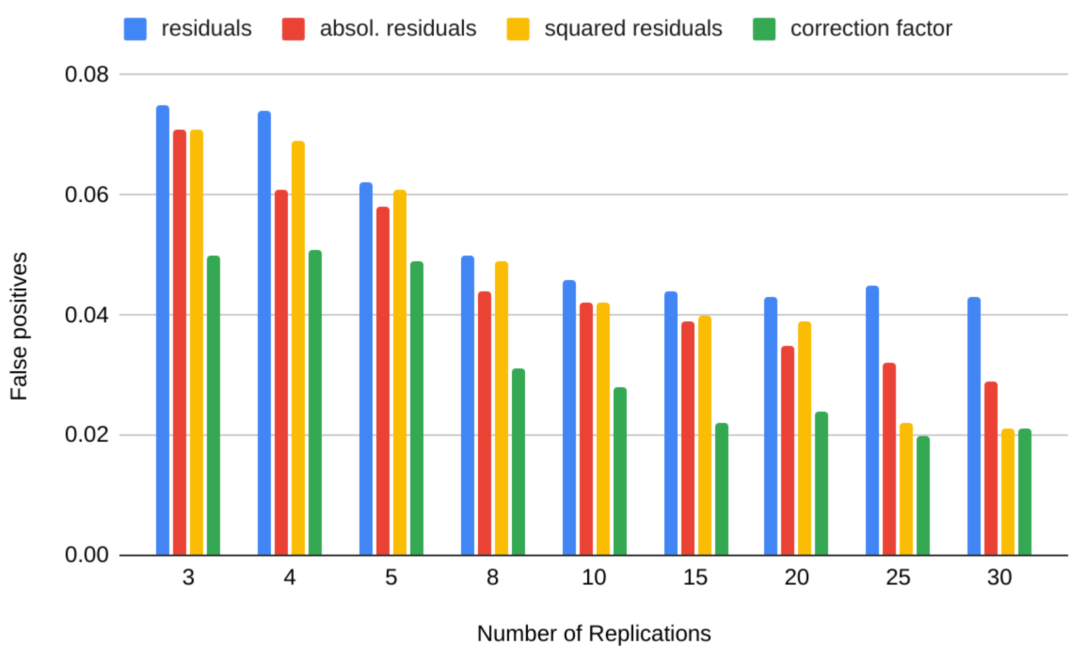

Figure 5: Comparison of false positives for higher order factorial ANOVA

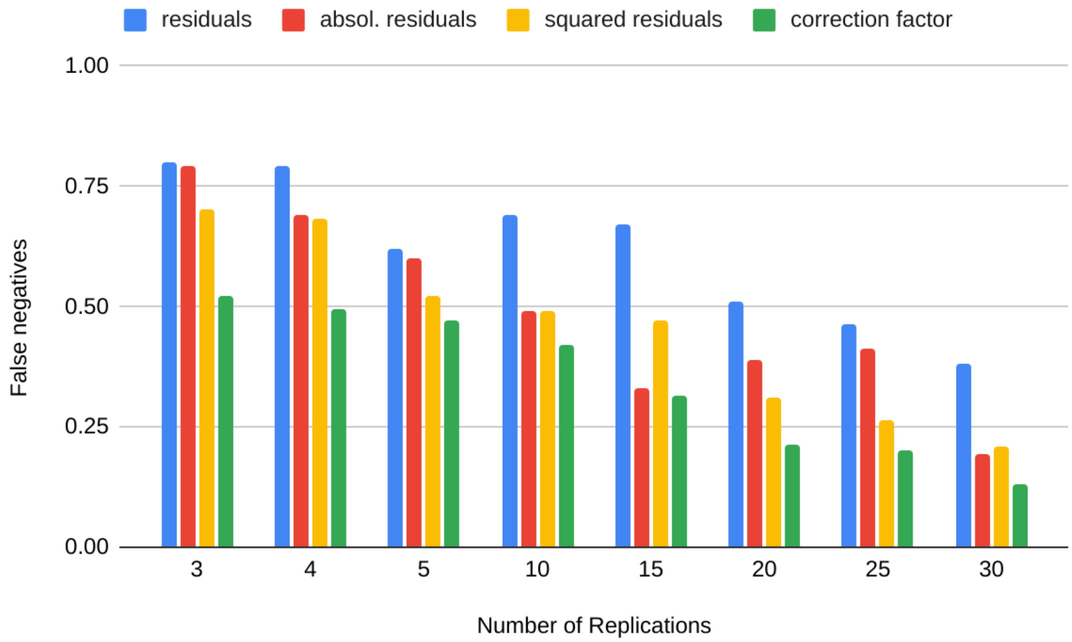

Figure 6: Comparison of false-negatives for higher order factorial ANOVA

Separate Analysis of Main and Interaction Effects using Levene's test

This particular scenario was created primarily to study the main impact and interaction outcomes for various residual types. hence, the residuals, the absolute residuals, and the square residuals, and with the correction factor.

4.0.4 Discussions

The findings showed that the test's interaction effect and main effects for variables A and B were both statistically significant. Using the residuals, it was determined that factor A was 0.048, factor B was 0.050, and the interaction effects (A*B) were 0.045 from the findings for false positives. Factor A was 0.040, factor B was 0.037, and the interaction effects (A*B) were 0.049 when the absolute residuals were taken into account. Additionally, factor A was 0.049, factor B was 0.047, and the interaction (A*B) was 0.042 according to the findings gathered via the square of the residuals. The findings for component A were 0.030, factor B was 0.024, and interaction effects (A*B) were 0.018 when employing the correction factor once more. An overview of false positives for the Levene test for three levels of factor A and two levels of factor B is presented in Table 5a. Again FIGURE. 7 demonstrates a comparison of false positives for 2*3 factorial trials with main and interaction effects. Three repetitions were performed using various residual representations and the correction factor. Factors A and B were 0.22 and 0.28 respectively for false negatives, whereas factor A*B (interaction effect) was 0.25. This demonstrates a statistical power using residuals of 0.78, 0.72, and 0.75 for components A, B, and A*B, respectively. Factor A was 0.18, factor B was 0.26, and factor A*B

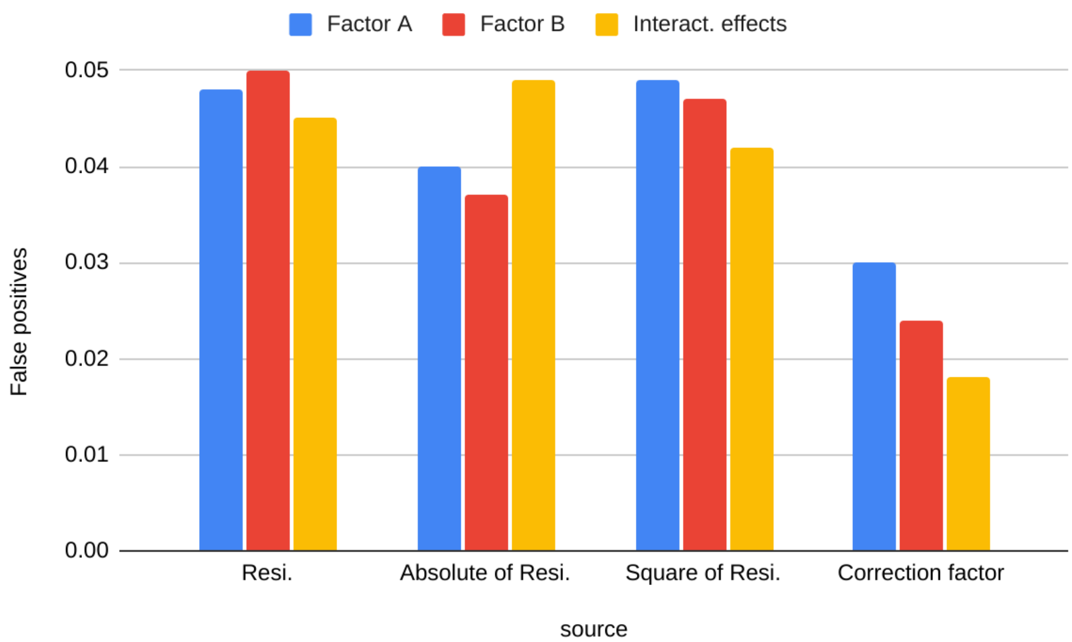

Figure 7: Comparison of false positives for 2*3 factorial trials with main and interaction effects

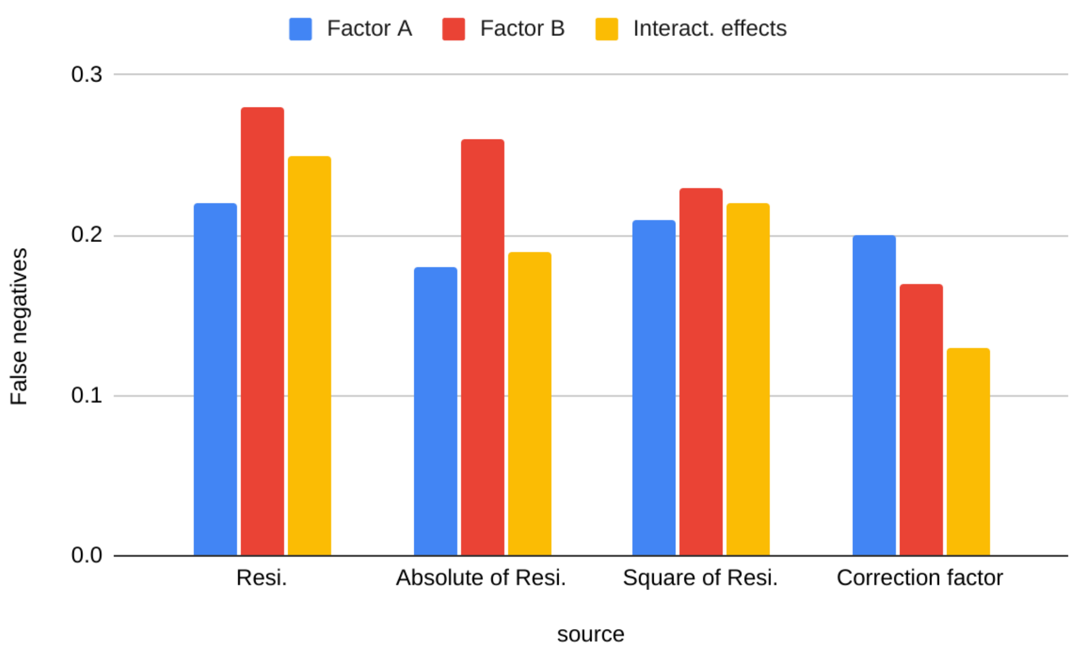

Figure 8: Comparison of false negatives for 2*3 factorial trials with main and interaction effects

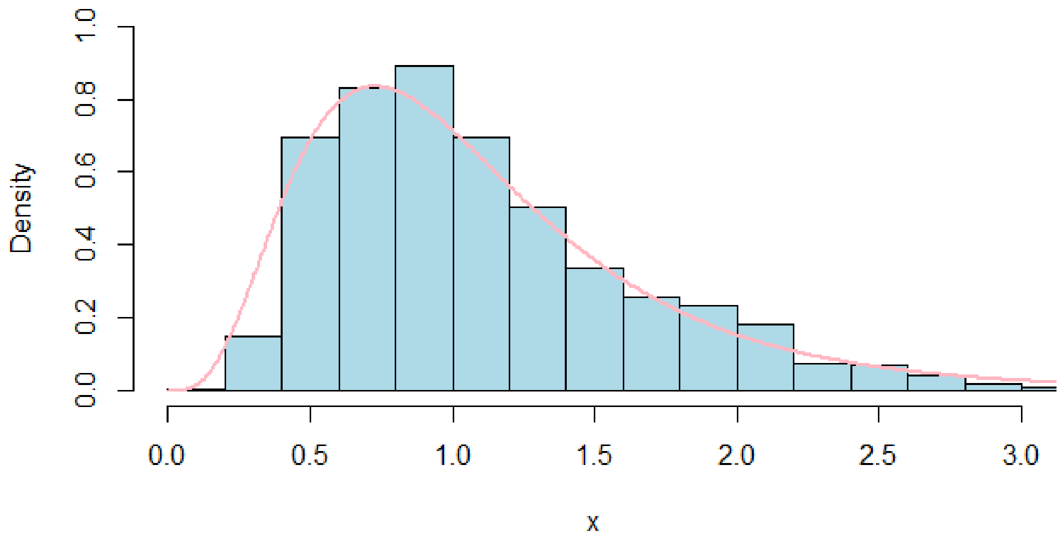

Figure 9: Cross-Validation of pattern for the fitted residual from the modified Levene's test with the F- distribution

(interaction effect) was 0.19 when considering the absolute residuals. For components A, B, and A*B, this demonstrates statistical powers of 0.82, 0.74, and 0.81. Additionally, factor A was 0.21, factor B was 0.23, and factor A*B (interaction effect) was 0.22 deploying the square of residuals. This demonstrates statistical powers of 0.79, 0.73, and 0.78 for factors A, B, and A*B, respectively. The findings for factor A were 0.20, factor B was 0.17, and interaction effects (A*B) were 0.13 when utilizing the correction factor once more. This demonstrates statistical powers of 0.80, 0.83, and 0.87 for factors A, B, and A*B, respectively, TABLE 5b demonstrates false negatives for the Levene test for three levels of factor A and two levels of factor B using three repetitions for each of the three different residual types. Also, FIGURE. 8 demonstrates a comparison of false negatives for 2*3 factorial trials with main and interaction effects. It is important to note that each residual form was tested under 1000 simulations for false positives and false negatives. Levene's test performs better for false positives and false negatives outcomes when employing the correction factor, as suggested in this study, and it provides the highest statistical power out of all the possibilities taken into account. This happens due to the fact that utilizing residuals, absolute residuals, the square of residuals, and the correction factor, respectively, the power of statistics for Levene's impact for interaction was 75%, 82%, 78%, and 87%. Once more, the results of this study outperformed those of the traditional Bartlett's and Levene's tests (Jayalath et al., 2017), Wang et al., 2017, Li et al. (2017), Odoi et al. (2019), Chiu et al. (2023) and several additional modified Bartlett's and Levene's tests that have been described in the literature. The issue of conservatism when using the traditional Levene's test is resolved by this enhanced Levene's test with the correction factor.

5. DISTRIBUTION CROSS-VALIDATION

To determine if the distributions obey the F-test, the easy-fit software program 3.0 was utilized for distribution fitting. With regards to degrees of freedom, the pattern is F- distribution FIGURE. 9 shows the pattern of the residuals of the fitted test with the F distributed $F_{\alpha,k-1,N-K}$.

6. CONCLUSIONS

In factorial ANOVA, choosing the optimal test to examine HOV is a tactical and strategic decision for researchers. However, it has been determined from this study that Levene's test, which was described by some statisticians as conservative to false positives and false negatives [5, 22, 24, 28, 29], when testing homogeneity of variance in factorial ANOVA, actually executes more effectively when using these different kinds of residuals and the correction factor as stated in this study. With a focus on the statistical strength and the requirement of false positives, this notion takes into account the number of levels and replicates. It is interesting to note that the accuracy of false positives and false negatives results for all five simulations increased as the number of replications rose. Again, it was observed from the simulation that the various residuals and correction factors taken into account for Levene's test under the varied number of levels and uniformity led to a significant boost for false positives and false negatives. As the number of levels and accuracy rose, false positives significantly decreased to an infinitesimal minuscule value. In addition, regardless of the number of factors, the statistical power was greatly enhanced when the correction factor was used with the absolute residuals. With an increase in the number of repetitions, the suggested test statistic for Levene's utilizing the various residuals and the correction factor decreases both false positives and false negatives. The suggested test statistic is therefore effective and reliable. Obviously, when using the proposed Levene's test with a correction factor, fewer repetitions at different levels may be needed to conduct research in order to minimize false positives and boost the test's statistical strength.

7. RECOMMENDATIONS

Researchers are advised to employ this strategy to assist address the issue of conservatism in assessing the homogeneity of variance in Factorial ANOVA. Again, the application of the developed novel should be explored in engineering, health, agriculture, and environmental and safety. Since this research is limited to CRD, other types of designs can be examined using the same principles.

References

[1] Arshad S, Khurram S. Survey Dataset on Citizens Perspective Regarding the Government's Use of Social Media for Provision of Quality Information and Citizens Online Political Participation in Pakistan. Data Brief. 2020;32:106311.

[2] Cochran WG, Cox GM. Experimental Designs. 2nd ed. New York: John Wiley Son. 1992:640.

[3] Cox DR. Principles of Statistical Inference. 3rd ed. 2006;237.

[4] Box GE. A General Distribution Theory of a Class of Likelihood Criteria. Biometrika. 1949;36:317-346.

[5] Odoi B, Samita S, Al-Hassan S, Twumasi-Ankrah S. Efficiency of Bartlett's and Levenes Tests for Testing Hov Under Varying Number of Replicate and Groups in One-Way Anova. Int J Innov Technol Explor Eng. 2019;8:1219-1223.

[6] Parra-Frutos I. Testing Homogeneity of Variances With Unequal Sample Sizes. Comp Stat. 2013;28:1269-1297.

[7] Chiu MC, Hwang GJ, Hsia LH. Promoting Students' Artwork Appreciation: An Experiential Learning□Based Virtual Reality Approach. Brit J Educational Tech. 2023;54:603-621.

[8] Alam A, Mohanty A. Evaluation of Software Engineering Virtual Laboratory in Determining Undergraduate Students Conceptual Understanding: A Blended Learning Model Using Collaborative-Creative Virtual Learning Environment Employing Critical Pedagogy. In: Sentiment analysis and deep learning: proceedings of ICSADL. Singapore: Springer Nature Singapore. 2023:875-899.

[9] Singh A, Kukreja V, Kumar M. An Empirical Study to Design an Effective Agile Knowledge Management Framework. Multimedia Tool Appl. 2023;82:12191-12209.

[10] Frenzel JE. Exploration of the Entrepreneurial Orientation and Trait Emotional Intelligence in Practicing Pharmacists. J Am Pharm Assoc. 2020;60:723-728.e2.

[11] Ahmad A, Akhsani L. The Effect of Blended Learning Type of Flipped Classroom on Mathematical Problem-Solving Ability in Solid Figures Topic. INOMATIKA. 2023;5:16-27.

[12] Wu Y, Zhou X, Qian M, Jin Y, Sun H, et. al. Novel Approach to Typical Air-Conditioning Behavior Pattern Extraction Based on Large-Scale Vrf System Online Monitoring Data. J Build Eng. 2023;69:106243.

[13] Ciklacandir S, Isler Y. Priority Assessment of Procuring Medical Equipment in Turkish Hospitals Using Input-Weighted Fuzzy Logic Architecture. Expert Syst Appl. 2023;213:119195.

[14] Box GE. Non-normality and Tests on Variances. Biometrika. 1953;40:318-335.

[15] Box GE, Anderson SL. Robust Tests for Variances and Effects of Non-normality and Variance Heterogeneity on Standard Tests. 1955;101:122.

[16] Mendeş M, Yiğit S. Type I Error and Test Power of Different Tests for Testing Interaction Effects in Factorial Experiments. Stat Neerl. 2013;67:1-26.

[17] Assaf OM, Abdulla SM. Air Quality Assessment in an Oil and Gas Field at Rush Area With Particular Reference to Sulfur Dioxide and Nitrogen Dioxide. Sci J Univ Zakho. 2023;11:59-64.

[18] Bartlett MS, Kendall DG. The Statistical Analysis of Variance Heterogeneity and the Logarithmic Transformation. Supplement to the J R Stat Soc. 1946;8:128-138.

[19] Bartlett MS. Properties of Sufficiency and Statistical Tests. Proc R Soc Lond A. 1937;160:268-282.

[20] Mirzababaei P, Moghadas Nejad FM, Hajikarimi P. Accelerated laboratory evaluation of the effect of deicing condition on fracture characteristics of silane-based modified asphalt mixtures. Theor Appl Fract Mech. 2023;125:103855.

[21] Carvalho AM, Mendes FQ, Borges PH, Kramer M. A Brief Review of the Classic Methods of Experimental Statistics. Acta Sci Agron. 2023;45:e56882.

[22] Carroll RJ, Schneider H. A Note on Levene's Tests for Equality of Variances. Stat Probab Lett. 1985;3:191-214.

[23] Levene H. Robust tests for equality of variances. In: Olkin I et al. editors. Contributions to Probability and Statistics: Essay in Honor of Harold Hotelling. Stanford CA: Stanford University Press. 1960:278-292.

[24] Jayalath KP, Ng HK, Manage AB, Riggs KE. Improved Tests for Homogeneity of Variances. Commun Stat Simul Comput. 2017;46:7423-7446.

[25] Wang Y, Rodríguez de Gil P, Chen YH, Kromrey JD, Kim ES,et. al. Comparing the Performance of Approaches for Testing the Homogeneity of Variance Assumption in One-Factor Anova Models. Educ Psychol Meas. 2017;77:305-329.

[26] Umlauft M, Konietschke F, Pauly M. Rank□Based Permutation Approaches for Non□Parametric Factorial Designs. Br J Math Stat Psychol. 2017;70:368-390.

[27] Romani SK, Yousif RR, Matrood HH. Analyzing the Factorial Experiments Variances With the Repeated Values in a Practical Application. Nonlinear Anal Theor Methods Appl. 2022;13:921-936.

[28] O'Brien RG. Robust Techniques for Testing Heterogeneity of Variance Effects in Factorial Designs. Psychometrika. 1978;43:327-342.

[29] Odoi B, Twumasi-Ankrah S, Samita S, Al-Hassan S. The Efficiency of Bartlett's Test Using Different Forms of Residuals for Testing Homogeneity of Variance in Single and Factorial Experiments-a Simulation Study. Sci Afr. 2022;17:e01323.

[30] Bhandary M, Dai H. An Alternative Test for the Equality of Variances for Several Populations When the Underlying Distributions Are Normal. Commun Stat Simul Comput. 2008;38:109-117.

[31] Box JF. The Life of a Scientist. New York: John Wiley Son; 1978:141.

[32] Sharma D, Kibria BM. On Some Test Statistics for Testing Homogeneity of Variances: A Comparative Study. J Stat Comput Simul. 2013;83:1944-1963.

[33] Yi Z, Chen YH, Yin Y, Cheng K, Wang Y, et al. Brief Research Report: A Comparison of Robust Tests for Homogeneity of Variance in Factorial Anova. J Exp Educ. 2022;90:505-520.

[34] Chang P, Zhao Z, Xu G, Ghosh A, Huang J, et. al. Evaluation of the Coal Dust Suppression Efficiency of Different Surfactants: A Factorial Experiment. Colloids Surf A Physicochem Eng Aspects. 2020;595:124686.

[35] Snedecor GW, Cochran WG. Statistical Techniques. 8th ed. Ames IA: Lowa Kingdom University Press. 1989:345.

[36] Wahid Z, Nadir N. Improvement of One Factor at a Time Through Design of Experiments. World Appl Sci J. 2013;21:56-61.

[37] Nguyen D, Kim E, Wang Y, Pham TV, Chen YH, et. al. Empirical Comparison of Tests for One-Factor Anova Under Heterogeneity and Non-normality: A Monte Carlo Study. J Mod Appl Stat Methods. 2020;18:2-30.

[38] Hesse CA, Ofosu J. B. Statistical Methods for Social Sciences. Akrong Publication Limited. 2017:460.

[39] Baker TB, Smith SS, Bolt DM, Loh WY, Mermelstein R, et. al. Implementing Clinical Research Using Factorial Designs: A Primer. Behave Ther. 2017;48:567-580.

[40] Nsowah-Nuamah NN, Thattil RO. Intermediate Design and Analysis of Experiments. rev. ed. Legon Accra: ACADEC Press. 2013:362.

[41] Thulin M. Modern Statistics With R. Eos Chasma Press. Vol. 580. ISBN 9789152701515; 2021.

[42] Li I, Chen YH, Wang Y, Rodríguez de Gil PR, Pham T, et al. Jmasm 47: Anova$_hov$: $ASasMacroforTestingHomogeneityofVarianceinOne - FactorAnovaModels(Sas).JModApplStatMethods.$2017; 16 : 506 – 539.